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Grade Level/Subject: Algebra II

Topic: Using a Matrix to Predict the Winner

Objectives (P.A.S.S.): Algebra II Standard 1.3a. Add, subtract, and multiply matrices to solve problems.

Introduction: We had already spent several days learning the basic operations with matrices; however, the book seemed to focus mostly on network problems which didn't seem to connect too well with most of the students. The book had made a reference to sports rankings and predictions, so I tried to decide on a method to show them this. My "up-to-date" problem was given as a warm up on one day along with their assignment on network problems, then we used a "Numb3rs" activity from last year which also brought in a sports matrix and a few new ideas on the following day. The "Numb3rs" activity also helped me show students how the graphing calculators could help them do the kind of problems I had them doing by hand all week.

Resources: http://www.weallusematheveryday.com/tools/waumed/home.htm activity pages from Nov. 11, 2006. "Air Hockey"

Instructional process: The baseball information gathered from the internet allowed me to put on the board the following problem. "In the last meetings of each of the American league playoff teams, we had the following results: New York beat Detroit and Minnesota, Detroit meat Oakland, Oakland beat New York, and Minnesota beat Detroit and Oakland. Find [A], [A][A], (which actually said A squared but I couldn't find a way to do that on here) and [A][A]+[A]. Then predict who will go to the World Series." As we started the next day they were given the attached worksheet and asked to read the problem situation and answer as many of the first 10 questions as they could and then we worked together with how to use the calculators to work matrix problems 11 through 13. Once that became clear we discussed most of the answers, but I did leave the rankings for them to do for themselves.

Closure: The worksheet was an especially good way to review them on the meaning of the different things they were doing to their matrices, especially when question 13 (the last one) asked them to do something they hadn't done before which was to weight the matrix for the direct dominance above the 2^{nd} stage dominance..

Assessment: I use a mixture of class participation and worksheet on this kind of day.

Modifications/Accommodations: Any kind of sports records of teams that have already played each other could be used. The amount of data given can have a great effect of the degree of difficulty of the problem. Football wasn't very practical for this time of year, but could be used later if more interest. Some students need more assistance than others

on calculator days, but there are usually other students anxious to share their skills with a neighbor.

Reflection: The 15-25 minute time was not very realistic in my classes. It did take most of the class time (at least 40 minutes) with less time for calculator practice than I had expected. I have found that putting the matrix unit this early has had a strange effect. Some students who had done little or nothing to this point, grasp this area well and will hopefully be encouraged to continue that response as we return to more familiar things; however, others have been the exact opposite claiming that they "don't get it" as I have tried to approach it in other ways to reach them and even trying to use other students to help as well.

 ***NUMB3RS* Activity: Air Hockey**

Episode: "Convergence"

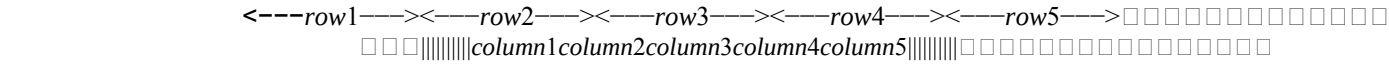
Topic: Matrices: Meaning and Operations **Grade Level**: 8 - 12
Objective: Introduction to how matrices can be used to organize data and solve problems.
Time: 15 - 25 minutes
Materials: graphing calculator

Introduction

A matrix is a rectangular array of numbers and/or variables. Matrices have been used to solve a variety of problems—from simple systems of equations to applications of game theory to analyzing complex communication networks. In "Convergence", random matrices and asymptotics are discussed. However, the goal of this activity is to introduce students to some basic matrix operations using wins and losses in a game to rank the players.

Discuss with Students

Let students answer questions 1 to 3 before you introduce the matrix used to rank players. You may discuss answers with your students and let them provide reasons for their rankings. You should discuss the differences between rows and columns of a matrix. Rows run across horizontally and columns run vertically:



Review with your students what the matrix represents. You may ask what the 1 in row 2, column 4 represents (Charlie beating Larry) or what the 0 in row 1, column 5 represents (Amita has beaten Megan 0 times). You could ask what would have to happen for a 2 to appear in the matrix (a player would have to beat another player twice).

You should review how to enter matrices into your calculator and how to perform operations with the matrices on the calculator before they attempt questions 11 to 13. If you have been teaching matrix multiplication you may want to explore why, when calculating $[A]^2$ in exercise 11, the result is the number of two-stage wins of a player (row) over another player (column).

Student page answers: **1***. Charlie. Both have two wins but Charlie beat Megan head to head.* **2***. Answers vary. Both have two wins but Charlie beat Megan who beat Don, so Charlie is favored over Don.* **3.** *Answers vary.* **4.** *Megan beat Don.* **5.** *Players may not play themselves.* **6.** *The row sums are: 0, 2, 2, 1, 2. Each row sum tells the wins for each player.* **7.** *The column sums are: 2, 0, 1, 2, 2. Each column sum tallies losses for each player.* **8.** *Yes; It would mean Amita beat Larry twice.* **9.** *Both the row sum, which represents the total number of wins and the column sum, which represents the total number of losses, add to 7. This number represents the total number of games played.* **10.** *Answers vary.* **11.** *The 1's represent the number of players that the row player has beaten who have beaten the column player. For example the 1 in row 2 (Charlie) and column 3 (Don) means that Charlie beat a player, namely Megan, who beat Don. This is called a "two-stage win."* **12.** *Row sums are 0, 5, 3, 3, 4. Ranking, if based on row sums would be 1: Charlie, 2: Megan 3: Don and Larry (tied) 5: Amita.* **13.** *1: Charlie, 2: Megan, 3: Don, 4: Larry, 5: Amita; The 2 gives a one-stage win twice as many points as a two-stage win.*

 NUMB3RS Activity Episode: "Convergence"
Name: _____ Date: _____

NUMB3RS Activity: Air Hockey

Amita, Charlie, Don, Larry, and Megan are avid air hockey players. Charlie decided to keep track of the results of the games that were played. Charlie beat Larry and Megan. Don beat Larry and Amita. Megan beat Amita and Don. Larry beat Megan in a spirited match. The five players decide to have a tournament but need to rank the players from best to worst before they begin to determine the brackets.

For exercises 1-13, use the information given in the example above.

1. Whom would you rank as a better player, Charlie or Megan?

_____

2. Use the results above to predict the winner of a game between Charlie and Don. _____

3. Rank all the players from 1 (best) to 5 (worst).

A matrix can be used to organize the data in the paragraph above. Matrix *A* below is a 5 X 5 matrix; it has 5 rows and 5 columns. The 1s in the matrix represent a win by the player in the row over the player in the column. The "1" in row 2, column 4 means that Charlie beat Larry.

4. What does the 1 in row 5, column 3 represent?

_____

5. Why are there only 0s along the diagonal of the matrix?

_____

6. A row sum is found by adding all of the entries in a row. The row sum of row 1 is 0. Calculate the row sums for the other 4 rows and explain what each sum represents.

_____

7. Calculate the column sums for all five columns and explain what each sum represents. _____

8. It is possible to have a 2, 3, 4, or any whole number in a matrix? If there were a 2 in row 1, column 4, what would this represent?

_____

9. Add all the row sums together. Do the same for the column sums. What do these two sums represent? _____

10. Use your matrix to re-rank the five players.

_____

11. Enter the matrix into your calculator and calculate the matrix A^2. You can do this in your TI-84 Plus calculator by entering [*A*] on the home screen, then pressing the $\mathbf{x^2}$ button and pressing [ENTER]. Look in row 2, column 1. This shows that Charlie beat Megan and Megan beat Amita. What does the number in row 2 column 3 represent?

_____

12. Charlie uses his TI-84 Plus calculator to calculate the sum of matrix [*A*] and matrix $[A]^2$. He uses the row sums to rank the players. What would his rankings be? _____

13. Don suggests a better ranking system using the row sums of $2[A] + [A]^2$. What would his rankings be? Explain the meaning of the 2 in this expression.

_____

The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Activity: Using your TI-84 Plus Calculator in Sports Predictions

Use your graphing calculator to enter an 11 X 11 matrix that represents the results of the current Big Ten (college football conference) results. Use the matrix and any appropriate operations to rank the teams and also predict results for next weekend's games.

Activity: Extra Thought

For the Student

- Use the matrix multiplication algorithm to explain why the matrix $[A]^2$ gives you the secondary wins or the number of wins for each team that you beat.

- Make up a 4 X 4 matrix [*B*] so that each of the four players is equally ranked when using row sums and column sums.

- Make up a 4 X 4 matrix [*C*] that has a number other than 0 in one of the entries in the diagonal of matrix $[C]^2$. Explain what this means.

Additional Resources

Graph and adjacency matrix:
http://www.cs.usask.ca/resources/tutorials/csconcepts/1999_8/tutorial/beginner/matrices/matrix.html
This site introduces the connection between a graph and an adjacency matrix. Links are provided to show how matrix multiplication is computed.
Adjacency matrices and graph theory:
http://mathworld.wolfram.com/AdjacencyMatrix.html
This site provides a definition of adjacency matrices as used in graph theory.
Alexei Borodin:
http://www.claymath.org/fas/research_fellows/Borodin/
This site introduces Alexei Borodin. His work on asymptotics and random matrices are highlighted in the lecture given during the episode.